# Assignment 1: Double Integrals

Be sure to show all work, not just the final answer. The assignment is due at the beginning of class, July 31st.

1. (a) Estimate the value of
$$\int_R yx^2$$
for $R = [0, 2] \times [0, 2]$ using $n = 4$, $m = 2$, and lower left endpoints.

   (b) Estimate the value of
$$\int_R yx^2$$
where $R$ is the region bounded by $y = x^2 - 1, y = 0, x = 0, y = 2$, using $n = 4$, $m = 2$, and lower left endpoints.

2. Evaluate the following double integrals:

   (a)
$$\int_0^2 \int_0^3 x - y \, dx \, dy$$

   (b)
$$\int_0^1 \int_1^2 ye^{xy} \, dy \, dx$$

   (c)
$$\int_0^4 \int_y^{\sqrt{y}} x^3 + 4y \, dx \, dy$$

   (d)
$$\int_D \cos y \, dA,$$
where $D$ is bounded by $y = x, y = -x, y = \pi/2$.

(e)

$$\int_D 1 - x^2 \, dA$$

where $D$ is the circle with radius 2.

3. Use a double integral to find the area of the shape bounded by the curves $x = e^y, x = \sin y, y = \pi$, and $y = -\pi$.

4. Use a double integral to find the area enclosed by one loop of the curve $r = \cos 2\theta$.

5. Find the volume of the shape bounded by $z = x^2 + 4, y = 4 - x^2, x + y = 2, z = 0$.

6. Find the volume of the solid which is inside the sphere $x^2 + y^2 + z^2 = 25$ and outside the cylinder $x^2 + y^2 = 9$ (polar co-ordinates will be helpful).

7. Suppose $A$ is a thin sheet of metal, which can be described as the region bounded by $x = 0, y = 0, x + y = a$. (In other words, $A$ is an isosceles triangle whose same side lengths are $a$). Suppose that the density of metal at a point $(x, y)$ is $k(x^2 + y^2)$. Find the centre of mass of $A$.